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ONE-DIMENSIONAL WATER VAPOR EXPANSION WITH CONDENSATION AT HIGHER PRESSURES

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Abstract—Numerical calculation of nucleating water vapor flow through a nozzle at high pressure is presented. Contrary to the classical approach which takes into account only latent heat release due to condensation, the vapor phase removal and change of vapor structure has also been taken into account. This more general treatment of condensing vapor is needed when the influence of condensation of vapors at higher pressure on gasdynamic behavior of the flow is considered. At higher pressure the radius of critical droplets can be comparable with the mean free path. This has to be taken into account when the equation of droplets' growth is derived. A modified set of equations using Hertz and Knudsen models for droplet growth is proposed.

INTRODUCTION

All the schemes for calculation of water vapor expansion with homogeneous condensation can be divided in three basic parts, i.e. (1) equations of conservation and thermodynamic description of the condensing gas, (2) an expression for the nucleation rate, and (3) equations for droplet growth. This division corresponds to the physical picture of the flow with condensation. Many attempts have been undertaken to improve the models used in the above analysis. In the present paper, attention will be focused on the state and caloric equation of water vapor and on the growth of nuclei at high pressure. The problem of proper thermodynamic presentation of condensing vapor has been discussed by Bakhtar *et al.* (1975) and Ludwig (1975). In these papers the virial coefficient of the first order was introduced in the state equation giving the local specific heat as a function of temperature and pressure. Attempts have been made to allow for real gas effects when condensation occurs at higher pressure. This is of importance for the nuclear power station technology where the condensing flow of steam in the turbine occurs at high pressure. From a theoretical point of view, it is interesting to find out how the condensation at higher pressure can influence the gasdynamic field and what are the characteristics of the condensing phase (i.e. concentration and sizes of droplets). Contrary to the situation at low pressure (below 1 bar) there are not very many experimental investigations at the higher pressure (up to 100 bars). A few experiments were carried out by Gyarmathy *et al.* (1973) and for intermediate values of pressure by Valha & Ryley (1977). These experiments give some idea of the differences between condensing steam at low and high pressure. The higher the pressure, the more dense is the condensing phase and the greater the size of the droplets. There was also evidence of a weak detonation type of a change in the gasdynamic behavior of condensing steam at higher pressure in these experiments.

According to the theoretical analysis by Puzyrewski (1975) for the steam at the pressures above 30 bars the latent heat released is lower than removal of gas enthalpy by condensing molecules. From this point of view the gasdynamic effect of condensation at higher pressure should be opposite to the one observed at lower pressures. The authors of the present paper have revised the problem of the gasdynamic behavior of condensing vapor in order to explain the experimental facts. It has been found that three physically important factors are responsible for the gasdynamic behavior of condensing vapor at higher pressure.

Two of them are included in the energy conservation equation, namely heat release and gas enthalpy subtraction from the flow. They are coupled in nondimensional parameter (L/C_pT_S) where L is latent heat, C_p is local specific heat and T_S is saturation temperature. The third one comes out of the equation of state. The structure of vapor different from the structure of ideal gas influences the state equation through the compressibility factor. This factor influences the behavior of gasdynamic field very strongly.

As the mean free path decreases when the pressure rises there is also a need to re-examine the formulae for nuclei growth. At low pressure the nuclei, or so called critical droplets, are small in comparison with the mean free path. So there is heat and mass exchange at the free molecular level.

At the high pressure the mean free path is of the order or smaller than the size of critical droplet so that the heat and mass exchange is partly of free molecular and partly of continuous character. This has to be taken into account when condensation is calculated for a broad range of pressure changes.

CONSERVATION EQUATIONS

We will confine our consideration to the steady one-dimensional model of condensing flow. There is a criticism of the validity of one-dimensional approach to the flow with condensation. It has been shown by Bratos & Meier (1976) that the two-dimensional model describes better the condensation front. Nevertheless, one dimensional approach is commonly used in engineering types of calculation and has in certain cases quite satisfactory accuracy. The general features of condensing flow can also be described in the frame of one-dimensional model.

Mass conservation

If the relative volume of condensing phase is small, the mass conservation equation may be written in the following form:

$$\frac{\rho_G}{1-y} uA = m_0 \tag{1}$$

where ρ_G is the density of gas, y is the wetness fraction, u is the velocity and A is the cross section of the nozzle. The more general form is

$$[(1-\vartheta_L)\rho_G + \vartheta_L \rho_L] uA = m_0 \tag{2}$$

which can be rearranged with the assumption that $\vartheta_L \ll 1$ and $y = \vartheta_L \rho_L / (\rho_G + \vartheta_L \rho_L)$ into [1] where ϑ_L is the relative volume of liquid, and ρ_L is the density of liquid.

Momentum conservation

Let us introduce the impulse function I

$$I(x) = m_0 u_0 + p_0 A_0 + \int_0^x p \frac{dA}{dx} dx \tag{3}$$

where x is the coordinate along the nozzle and p the pressure. Then the momentum conservation equation may be rewritten in the form:

$$mu + pA = I(x). \tag{4}$$

This is an integral type of equations with respect to p.

Energy conservation

The steady flow energy conservation equation for the condensing vapor has the form:

$$\frac{u^2}{2} + h = h_0 \tag{5}$$

where h is the enthalpy of condensing mixture.

If the saturation conditions are used as the reference, the enthalpy of condensing vapor can be written as:

$$h = (1-y)\left(h_{GS} + \int_{T_S}^{T_G} C_p \, \mathrm{d}T\right) + y\left(h_{LS} + \int_{T_S}^{T_L} C \, \mathrm{d}T\right) \quad [6]$$

where h_{GS}, h_{LS} are the enthalpies of saturated gas and liquid and T_G, T_L, T_S temperatures of gas, liquid and saturation temperature.

It is convenient to introduce the mean specific heat

$$\bar{C}_p = \frac{1}{T_G - T_S} \int_{T_S}^{T_G} C_p \, \mathrm{d}T \quad [7]$$

$$\bar{C} = \frac{1}{T_L - T_S} \int_{T_S}^{T_L} C \, \mathrm{d}T. \quad [8]$$

The relation [6] can be rewritten in algebraic form

$$h = (1-y)[h_{GS} + \bar{C}_p(T_G - T_S)] + y[h_{LS} + \bar{C}(T_L - T_S)]. \quad [9]$$

The latent heat L is defined at saturation conditions as

$$L = h_{GS} - h_{LS}. \quad [10]$$

Introducing [9] and [10] into energy equation we get

$$\frac{u^2}{2} + (1-y)\bar{C}_p T_G - \left[1 - \frac{\bar{C}_p T_S}{L} - \frac{\bar{C}(T_L - T_S)}{L}\right] Ly - \bar{C}_p T_s + h_{GS} = h_0. \quad [11]$$

It is noteworthy that from [11] follows the approximation commonly used for the calculation of condensing flow at low pressure.

When $(\bar{C}(T_L - T_S)/L)$ plus $(\bar{C}_p T_S/L)$ is small compared to unity and

$$h_{GS} = \bar{C}_p T_S + \text{const.}$$

which is a good approximation for low pressure and also $y \ll 1$, then [11] becomes:

$$\frac{u^2}{2} + \bar{C}_p T_G - Ly = \text{const.} \quad [12]$$

The form [12] of energy equation is justified at low pressure. It takes into account only heat release due to condensation.

EQUATION OF STATE

To close this set of conservation equations the equation of state is needed.

We will apply the form

$$\frac{p}{\rho_G T_G} = RZ(p, T_G) \quad [13]$$

where Z is so called the compressibility factor and R gas constant.

This form of state equation is very convenient for describing the real properties of steam given in steam tables.

For different types of Z function different models of processes can be discussed.

From the above set of equations, the basic quadratic form for velocity can be derived.

$$u^2 - 2\frac{\bar{C}_p}{2\bar{C}_p - RZ(p, T_G)}\frac{I(x)}{m_0}u + \frac{2RZ(p, T_G)}{2\bar{C}_p - RZ(p, T_G)}\left\{\left[1 - \frac{\bar{C}_p T_S}{L} - \frac{\bar{C}(T_L - T_S)}{L}\right]Ly + h_o - h_{GS} + \bar{C}_p T_S\right\} = 0. \qquad [14]$$

This is a convenient form to discuss the gasdynamic behavior of the solution at the throat (saddle point) and the influence of condensation as well.

CHARACTERISTIC FEATURES OF THE QUADRATIC FORM[14]

Two important parameters are included in the quadratic form[14] namely m_0 and $Z(p, T_G)$.

The mass flow rate m_0 plays an important role in the structure of the saddle point at sonic conditions. If the mass flow rate m_0 is chosen properly at every coordinate x two roots of[14] exist. At the throat a double root exists

$$u_* = u_1 = u_2 = \frac{\bar{C}_p}{2\bar{C}_p - RZ(p, T_G)}\frac{I(x_*)}{m_0} = \sqrt{\left(\frac{2RZ(p, T_G)}{2\bar{C}_p - RZ(p, T_G)}\left\{\left[1 - \frac{\bar{C}_p T_S}{L} - \frac{\bar{C}(T_L - T_S)}{L}\right]Ly + h_0 - h_{GS} + \bar{C}_p T_S\right\}\right)} \qquad [15]$$

and this a singular point.

An error in the mass flow rate m_0 changes the structure of the solution at the singular point.

It is noteworthy that while applying the quadratic form of integral type[14] there is no need to differentiate all the parameters describing the behavior of real gas.

The compressibility factor Z and the mean specific heat can be expressed in the form of polynomials to approximate the steam tables. The influence of condensation appears in the equation as

$$\left[1 - \frac{\bar{C}_p T_S}{L} - \frac{\bar{C}(T_L - T_S)}{L}\right]Ly. \qquad [16]$$

If we assume that the temperature of liquid T_L phase does not exceed saturation temperature considerably, the term $(\bar{C}(T_L - T_S)/L)$ can be neglected. The condensation factor then is reduced to

$$\left(1 - \frac{\bar{C}_p T_S}{L}\right)Ly. \qquad [17]$$

The influence of this parameter has been discussed by Puzyrewski (1975). The discussion presented there is valid for either ideal or isentropic process of condensation, i.e.

$$Z(p, T_G) = \text{const.} \qquad [18]$$

$$Z = Z(p). \qquad [19]$$

A more exhaustive numerical analysis presented in this paper lead the authors to the conclusion that neither the assumption[18] nor [19] is a good approximation in the real vapor condensing at higher pressure.

In spite of the fact that above 30 bars for steam $(\bar{C}_p T_S/L)$ is higher than unity and [17] has negative sign, the compressibility factor $Z(p, T_G)$ cancels this influence of negative factor in the cube brackets in[14]. The ratio $(\bar{C}_p T_S/L)$ represents the thermodynamic potential $\bar{C}_p T_S$ substracted by condensing molecules to the heat L added due to the condensation. From this point of view the condensation may appear as the heat addition Ly and enthalpy of vapor subtraction $\bar{C}_p T_S y$ depending on the sign of the expression[17]. The third factor which influences the solution[14] is:

$$\frac{2RZ(p, T_G)}{2\bar{C}_p - RZ(p, T_G)}. \qquad [20]$$

Replacing the temperature by the entropy s, we have

$$Z(p, T_G) = Z(p, T_G(s, p)) = Z(s, p).$$

The condensation process is not isentropic so for s increasing towards s_s, $Z(p, s)$ also increases and then the factor[20] increases. It is noteworthy, that the gas phase changes its entropy considerably when temperature rises from supercooling temperature to saturated temperature.

The value of $Z(p, T_G)$ depends on the number of monomers, dimers, trimers and so on in gas phase structure. This influences the equation of state. The factors[17] and [20] act in an opposite way at higher pressure. The resulting effect may be established by the numerical calculation for a given range of parameters.

CONDENSATION PROCESS

The nonequilibrium condensation appears in the quadratic form[14] as a wetness fraction y and the liquid and gas temperature T_L and T_G differs from the saturation temperature T_S. The wetness fraction is defined by the relative volume ϑ_L which can be calculated as:

$$\vartheta_L = \frac{4}{3}\pi\rho_G(x)\int_0^x \frac{\mathcal{T}(x')}{\rho_G(x')} r^3(x', x)\,\mathrm{d}x' \qquad [21]$$

where ρ_G is the density of gas, $\mathcal{T}$ the nucleation rate and r is the radius of droplets.

The radius of a droplet starts to grow from the critical size $r_*(x', x)$ formed at a position x' so we have:

$$r(x', x) = r_*(x', x) + \int_{x'}^x \frac{\mathrm{d}r}{\mathrm{d}x}\,\mathrm{d}x. \qquad [22]$$

The general formula for the critical radius is

$$r_* = \frac{2\sigma_\infty(T_G)}{\rho_L[\mu_G(p, T_G) - \mu_L(p, T_G)]} \qquad [23]$$

where the difference of thermodynamic potentials μ_G of gas phase and μ_L of liquid phase

$$\mu_G(p, T_G) - \mu_L(p, T_G) \qquad [24]$$

has to be taken from the tables at high pressure (σ_∞ means surface tension coefficient).

The classical expresion of Volmer–Frenkel type has been used for nucleation rate. In the most simple way it may be written as

$$\mathcal{T} = \frac{n}{\tau}\exp\left\{-\frac{4\pi r_*^3\sigma_\infty(T_G)}{3kT_G}\right\} \qquad [25]$$

where $n = (p/kT_G)$ [k—Boltzmann constant] is the concentration of single molecules and τ has the dimension of time which has been estimated as

$$\tau = \left(\Gamma\theta \frac{1}{\rho_L} \sqrt{\left(\frac{2m\sigma_\infty(T_G)}{\pi}\right)} \frac{p}{kT_G}\right)^{-1}. \qquad [26]$$

The adjustable parameter Γ has been evaluated by Barschdorff (1976) to remove the discrepancy between the theory of nucleation of Volmer–Frenkel type and the experimental results for the pressure range up to 30 bars. The coefficient θ has been introduced by Feder *et al.* (1966). It takes into account the nonisothermal effects.

For the growth of nuclei we will follow the Hertz and Knudsen model of condensation upon a liquid surface. The main criticism of this model is that the accomodation coefficients introduced there, especially the condensation coefficient α_c, have been determined between 0 and 1 in different experiments. Because of this large scatter this model has been considered to be very uncertain and it has been replaced by the diffusion models. Nevertheless, the Hertz and Knudsen model comes from a very reasonable consideration of the physical mechanisms at the interface between the droplet surface and the surrounding vapor in the free molecular regime.

The mechanisms of mass and heat transport are essential in the process of molecules condensation upon the surface of droplets. These two mechanisms are coupled. The heat transport is due to the molecules reflected and evaporated from droplet surface. The condensation heat removed from the droplet surface through temperature jump must be then transported from the neighbourhood of the droplet surface to the bulk of gas by the heat conduction mechanism (see Fig 1).

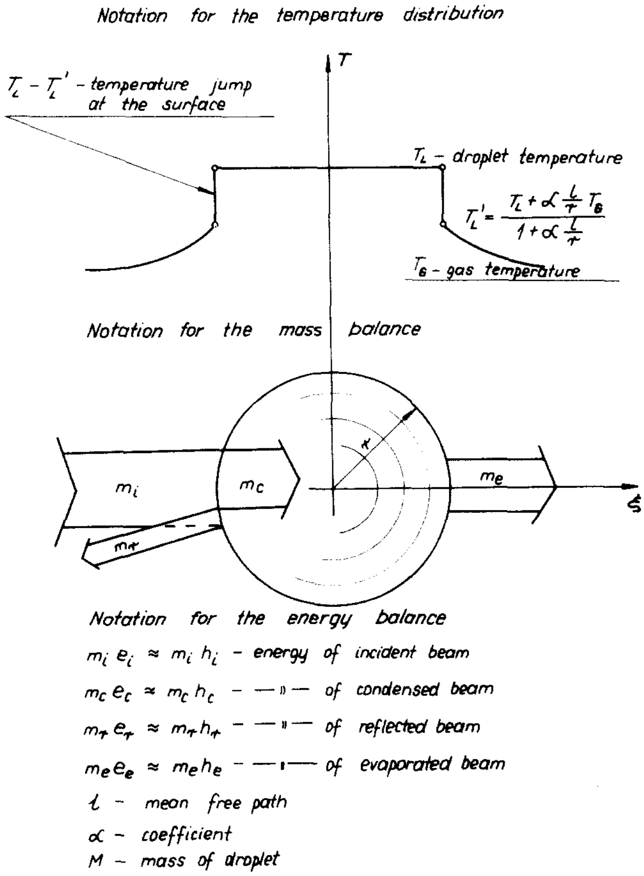

Figure 1. Notation for the model of droplet growth.

The Hertz-Knudsen model of condensation relates to the situation where the temperature jump between the surface and the vapor controls the mass and heat transport processes. A more general situation is when the temperature jump and the gradient of temperature are present at the surface. The temperature jump at solid surface according to the kinetic theory is of the order

$$\delta T = T_L - T'_L = \alpha l\left(\frac{\partial T_G}{\partial \xi}\right)_{\xi=r} \qquad [27]$$

where l is the mean free path and the coefficient α is of the order unity.

Let us assume that the estimation[27] is also valid for a condensing surface. Assuming, furthermore, that the steady state heat conduction from the droplet is an admissible approximation, the temperature gradient at the surface is then:

$$\left(\frac{\partial T_G}{\partial \xi}\right)_{\xi=r} = -\frac{T'_L - T_S}{r} \qquad [28]$$

From the above we have

$$\delta T = T_L - T'_L = \alpha\frac{l}{r}(T'_L - T_G) \qquad [29]$$

and

$$T'_L = \frac{T_L + \alpha\frac{l}{r}T_G}{1+\alpha\frac{l}{r}}. \qquad [30]$$

Neglecting the effusion effect we can approximate the flux of energies in the terms of enthalpies. According to the notation in figure 1 the energy balance for the droplet enthalpy is:

$$\frac{\mathrm{d}}{\mathrm{d}t}(Mh_L) = m_c h_c(T'_L) + m_r h_c(T'_L) - m_r h_r(T_L) - m_e h_e(T_L). \qquad [31]$$

Taking into account the mean specific heats[7] and[8] the above equation can be rearranged

$$M\frac{\mathrm{d}h_L}{\mathrm{d}t} = [L - (\bar{C} - \bar{C}_p)(T_L - T_S)]\frac{\mathrm{d}M}{\mathrm{d}t} - (m_c + \alpha_a m_r)\bar{C}_p(T_L - T'_L) \qquad [32]$$

where α_a is the thermal accomodation coefficient.

Comparing the heat conducted across the temperature jump (the second term on the r.h.s. of[32]) with the heat conduction due to the temperature gradient of the surface we get

$$(m_c + \alpha_a m_r)\bar{C}_p(T_L - T'_L) = 4\pi r^2\lambda\frac{T'_L - T_G}{r}. \qquad [33]$$

Rearrangement gives

$$\frac{2\alpha[\alpha_c + \alpha_a(1-\alpha_c)]}{2-\alpha_c}\frac{mp\bar{C}_p}{\sqrt{(2\pi mkT'_L)}}l = \lambda \qquad [34]$$

where α_c is condensation coefficient, l mean free path and λ is heat conduction coefficient.

The relation[34] may be helpful for the evaluation of condensation coefficient α_c when α and the thermal accomodation coefficient α_a are known. These three coefficients are tied together by relation[34]. The other models which are based on diffusion or selfdiffusion phenomena encounter the same type of difficulties when the magnitudes of temperature jump and the selfdiffusion coefficient are concerned.

Following the derivation presented by Puzyrewski & Krol (1976) and taking into account the above modifications the set of equations for droplets growth may be written as:

$$\frac{\mathrm{d}r}{\mathrm{d}t}=\frac{2\alpha_c}{2-\alpha_c}\frac{m}{\rho_L\sqrt{(2\pi mk)}}\left[\frac{p}{\sqrt{(T'_L)}}-\frac{p_S(r,T_L)}{\sqrt{(T_L)}}\right] \qquad [35]$$

$$\frac{\mathrm{d}T_L}{\mathrm{d}t}=\frac{3[L-(\bar{C}-\bar{C}_p)(T_L-T_S)]}{r\bar{C}}\frac{\mathrm{d}r}{\mathrm{d}t}-\frac{3\lambda}{r^2\bar{C}\rho_L}\frac{T_L-T_G}{1+\alpha l/r}. \qquad [36]$$

This is a more general model of condensation in the sense that it covers the range of small and large droplets in respect of the mean free path.

In this model of condensation coefficients Γ and $\alpha-s$ allow us to match the onset of condensation and the pressure disturbance due to condensation with experimental results. According to the published papers on condensation the measurements of pressure distribution and light scattering effects are predictable by this type of model. It is true that this comparison has been done for rather low pressure. One of the purposes of the present paper is to modify this model of condensation for high pressure.

THE NUMERICAL SCHEME

The scheme of calculation of the water vapor expansion in the Laval nozzle has been worked out on the basis of the described model. The scheme has to meet certain requirements, namely:

(1) To be convergent with respect to the integral term in[14].

(2) To allow to jump over the singular point at the throat.

(3) To couple the set of conservation equations with the set of equations describing the condensing process. Assuming that we are restricted to condensation in the supersonic part of the nozzle there is no upward influence of condensation upon the flow. In such a case "step by step" method can be applied.

The scheme of Picard type has been applied for iteration of integral term $I(x)$. For a given step n along x-axis an interative loop in respect to an improved value of I_n was repeated

$$I_n=I_n^{i-1}-p_n^{i-1}\frac{\Delta A_n}{2}+p_n{}^i\frac{\Delta A_n}{2} \qquad [37]$$

until the given accuracy

$$|p_n{}^i-p_n^{i+1}|<\epsilon_1$$

was achieved.

In the region where no condensation takes place the set of conservation equations has the form

$$u_n^{i2}+\Phi(p_n{}^i,T_n{}^i)\frac{I_n}{m_0}u_n{}^i+Q(p_n{}^i,T_n{}^i)=0 \qquad [38]$$

$$p_n{}^i=\frac{I_n{}^i-m_0u_n{}^i}{A_n} \qquad [39]$$

$$T_n{}^i=\frac{p_n{}^i(I_n{}^i-p_n{}^i\cdot A_n)\cdot A_n}{m_0{}^2(1-y)\cdot R\cdot Z(p_n{}^i,T_n{}^i)}. \qquad [40]$$

Such a scheme provides rather a good convergence.

The iterative procedure has been repeated between the inlet of the nozzle and the throat until the proper value of the mass flow rate was achieved. The behavior of solution near the saddle point is very sensitive to the error of mass flow rate and to the accuracy of $p_n{}^i$ and $T_n{}^i$ in the iterative loop i. Even if this error is small it is difficult to obtain two roots of [14] close enough to each other at the throat. By means of the linear extrapolation of coefficients in quadratic form [14] the sensitivity at the throat with respect to errors has been reduced.

$$\Phi(p_n{}^i, T_n{}^i) = \Phi(p_{n-1}, T_{n-1}) + \vartheta_n^{i-1}(p_{n-1} - p_n{}^i) \tag{41}$$

$$Q(p_n{}^i, T_n{}^i) = Q(p_{n-1}, T_{n-1}) + \psi_n^{i-1}(p_{n-1} - p_n{}^i) \tag{42}$$

where

$$p_n{}^i = \frac{I_n^{i-1} - m_0 u_n{}^i}{A_n}. \tag{43}$$

A few iterations in the subsonic part of the nozzle were quite enough to reduce the error in mass rate to ± 0.1 per cent and then jump to the supersonic part where condensation starts. The coupling procedure of two sets of equations have not been solved from the theoretical point of view quite satisfactorily. If the condensation takes place the parameter y is involved in [14]. In order to find the value of y we need to solve the system:

$$\frac{\mathrm{d}r}{\mathrm{d}t} = f_1(p_n, T_n, \ldots)$$

$$\frac{\mathrm{d}T_L}{\mathrm{d}t} = f_2(p_n, T_n, \ldots)$$

$$\mathcal{T}(x) = \frac{n}{\tau}\exp\left\{-\frac{4\pi r_*{}^3\sigma_\infty(T_G)}{3kT_G}\right\}$$

$$\vartheta_L = \frac{4}{3}\pi\rho(x)\int_0^x \frac{\mathcal{T}(x')}{\rho(x')} r^3(x', x)\,\mathrm{d}x'$$

$$y = \frac{\vartheta_L \rho_L}{\rho_G + \vartheta_L \rho_L}$$

which is closed if p_n and T_n are given. On the other hand p_n and T_n can be obtained from the conservation equation if y is calculated. To break this dependence the extrapolated value of y has been used to solve the set of conservation equations and afterwards y has been recomputed from the above system.

When the extrapolated and computed values of y fulfilled the condition

$$|y_{\text{extr}} - y_{\text{recalculated}}| < \epsilon_2.$$

the solution has been considered as being satisfactory.

RESULTS OF CALCULATION

The influence of the compressibility factor Z in the state equation on the solution is shown in figure 2. If we assume that Z is either constant or is the function of pressure only we get for high pressure the deviation from the frozen flow due to condensation, as indicated by the dotted line in figure 2. It is a characteristic feature that the condensation in this case decreases pressure compared with frozen flow and increases the velocity, contrary to the classical gasdynamic behavior with the heat release only. It can be explained by the fact that the vapor enthalpy subtraction is relatively high compared with the heat release L.

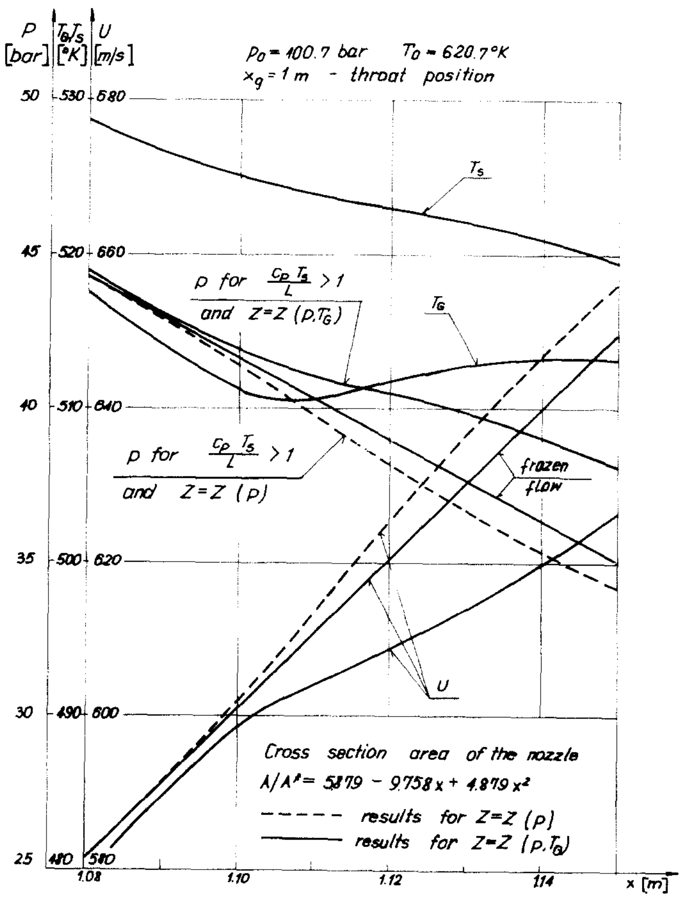

Figure 2. The influence of factor $(\tilde{C}_p T_S/L)$ and Z on the solution of [14].

The factor $Z(p, T_G)$ cancels this influence giving the spread out jump of pressure (full lines in figure 2).

In order to determine how the proposed scheme works at different pressures the calculations have been done for different experiments. The data was taken from references by Gyarmathy *et al.* (1974) and Bakhtar *et al.* (1975.) The nozzle area and pressure distribution was taken from the publication with figures in a small scale which reduces their accuracy. One of the problems in these calculations is to match the measured and calculated pressure distribution ahead of the condensation zone. It has been found in different experiments by Bakhtar *et al.* (1975), Ludwig (1975) and Puzyrewski (1969) that measured pressure distribution along the nozzle before condensation may be obtained from one dimensional calculation when geometrical cross section of the nozzle is corrected. There are a few reasons for these corrections namely: possible leakage of the vapor along the nozzle profile, three dimensional character of flow field and friction at the nozzle walls. The experiments taken for the sake of comparison with the calculations are indicated in Table 1. The results of comparison of pressure distribution

Table 1.

	Author's experiments	Gyarmathy *et al.* experiments	Bakhtar *et al.* experiments
Initial pressure & temperature level	$p_0 = 2.96$ bar $T_0 = 457.16$ K	$p_0 = 40.43$ bar $T_0 = 575.3$ K $p_0 = 100.7$ bar $T_0 = 620.70$ K	$p_0 = 21.84$ bar $T_0 = 516$ K $p_0 = 35.69$ bar $T_0 = 562.6$ K
Correction the nozzle area	Correction was needed	No correction	Correction was needed

and other calculated parameters of the flow are shown in figures 3–7. All the data concerning the vapor state as $\bar{C}_p$, $Z(p, T_G)$, etc. were taken from steam tables by Vukalovitch (1969).

According to the results shown in these figures the satisfactory agreement can be obtained with the experimental data, concerning the pressure distribution. Nevertheless, the correction of the nozzle shape still remains an open problem. Due to different conditions of the experiments there is no rational recipe how to correct geometrical cross section of the nozzle in order to match the measured pressure distribution ahead of condensation. This is the first uncertainty of a general character for the nozzle flow calculation.

The essential factor for the placing of the onset of condensation is the nucleation ratio. The idea of adjusting the classical theory of nucleation with the experiments, by means of correction parameter Γ seems to be reasonable. Unfortunately no unique answere can be found so far in respect to the value of this parameter at the high pressure. Among five different experiments analysed in this paper three of them indicate the value of Γ order 10^{-2} (figures 3, 6 and 7) whereas two experiments indicate the value of 10^2 (figures 4 and 5). This is the second uncertainty of the calculation method of the vapor flow with condensation. There is no clear idea what is the reason for the scatter of the coefficient Γ.

For the high pressure in Gyarmathy's experiment (1974) a good agreement with the calculated and measured droplet radius along the nozzle has been found. It is shown in figure 8. This agreement indicates that the proposed model of a droplet growth can be adjusted to the experiment. The droplet growth influences the shape of pressure distribution at the onset of condensation. Calculated pressure distribution at this point fits the experiment quite well.

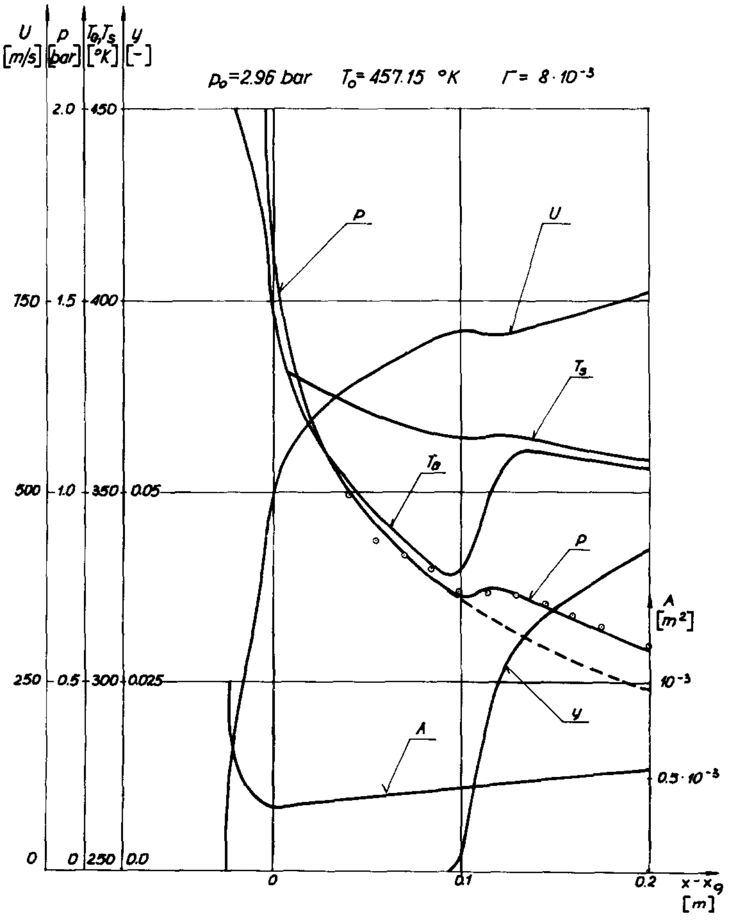

Figure 3. Calculated parameters and measured pressure at low pressure according to author's experiments.

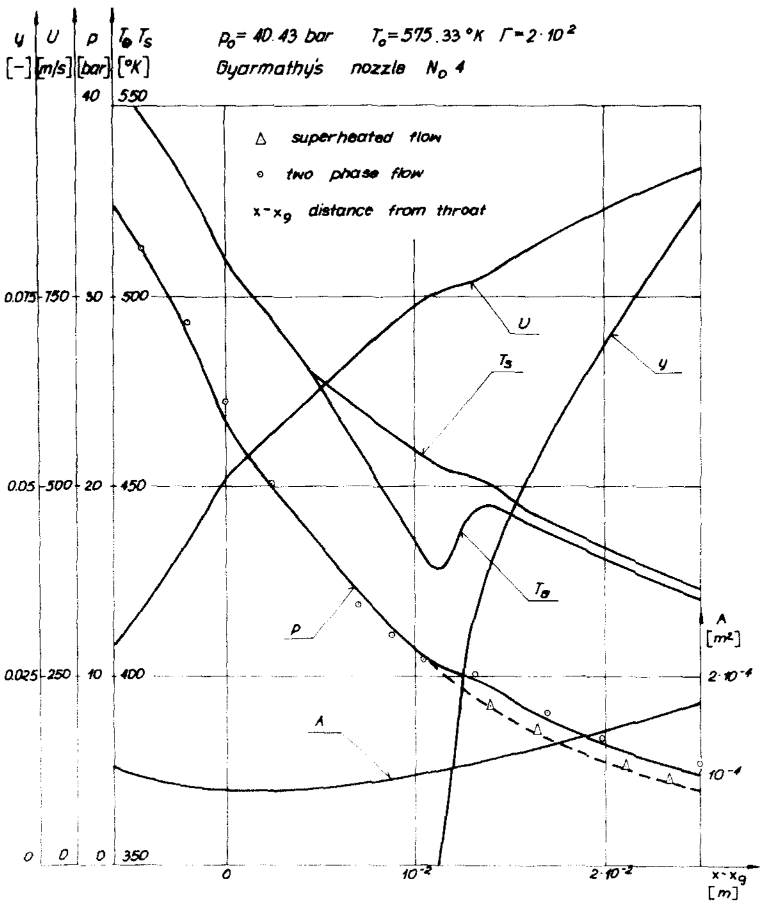

Figure 4. Calculated parameters and measured pressure at medium pressure according to Gyarmathy *et al.* (1974).

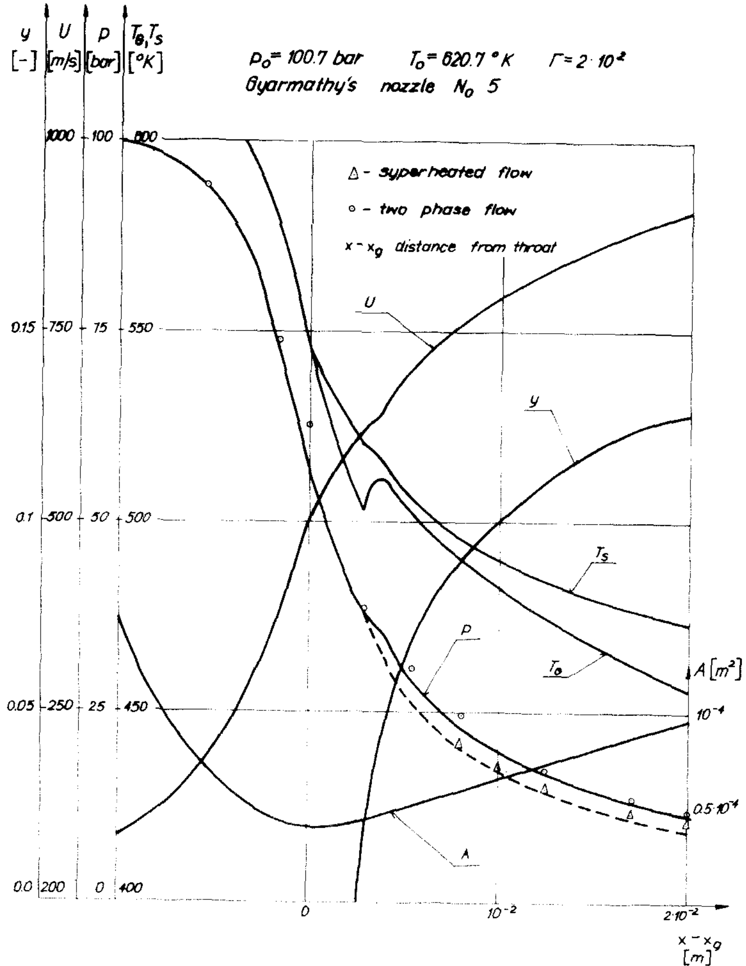

Figure 5. Calculated parameters and measured pressure at high pressure according to Gyarmathy *et al.* (1974).

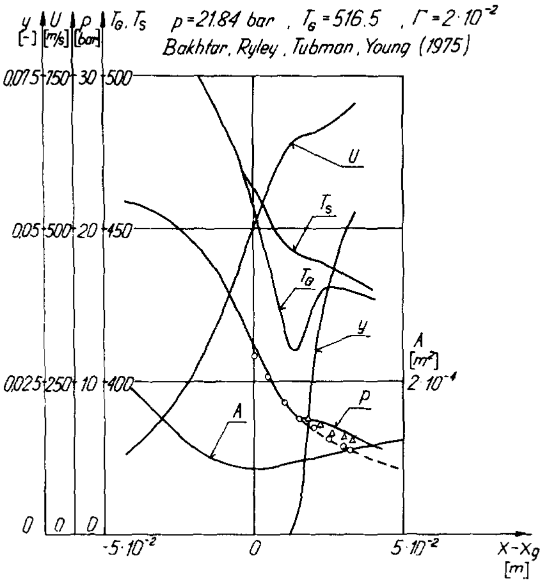

Figure 6. Calculated parameters and measured pressure at medium pressure according to Bakhtar *et al.* (1975).

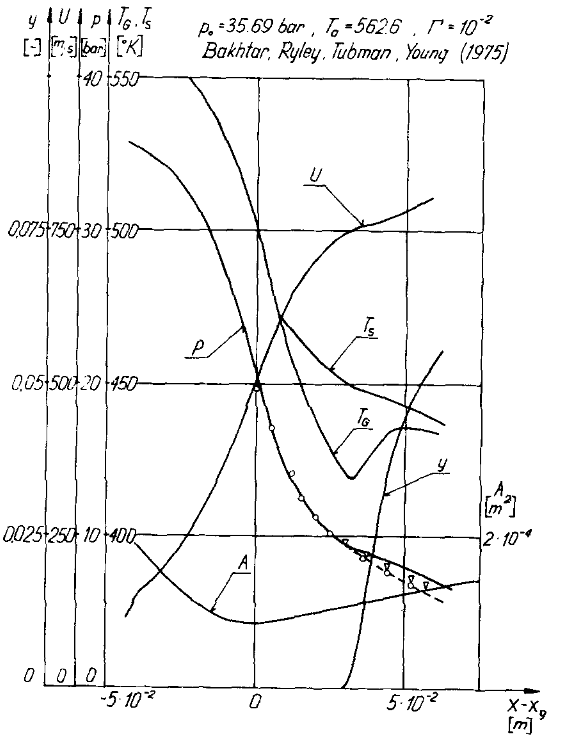

Figure 7. Calculated parameters and measured pressure at medium pressure according to Bakhtar *et al.* (1975).

The present model of calculation of the water vapor flow with condensation seems to agree with the real picture only qualitatively.

This conclusion can be also drawn out in the case of other models cited in the references. However, the matching factors in each model play an important role still.

The improvement of the experimental investigations and the improvement of nucleation theory has to be done in order to achieve better quantitative agreement between the theory and experiment.

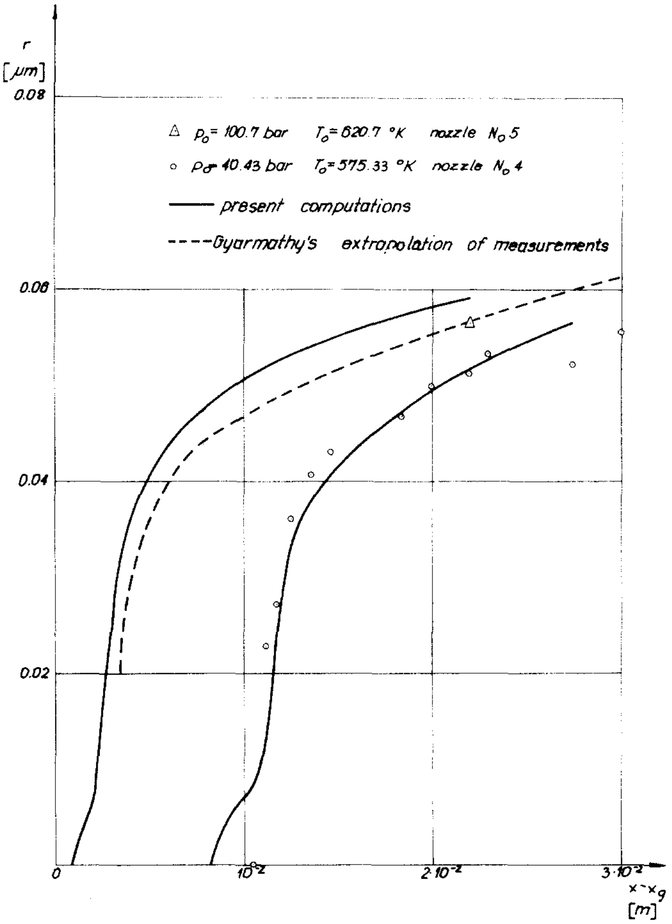

Figure 8. Calculated and measured mean radius of droplets according to Gyarmathy *et al.* (1974).

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